

B.Sc. Semester - 2 (NEP-2020) Examination

MARCH/APRIL-2025

MATH: Partial Derivatives & Differential Geometry (Major-4)

Time: 2:00 Hours

Marks: 50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

- Que-1(A) Answer any one question out of two. (05)
- (1) if function $Z = f(x,y)$ possesses the first order partial derivatives in the Domain D and $x=g(t)$, $y=h(t)$ also possesses its first order partial derivatives then show That $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$.
 - (2) if $u = f(z)$ and $z^2 = x^2 + y^2$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(z) + \frac{1}{z} f'(z)$.
- Que-1(B) Answer any one question out of two. (05)
- (1) State young's theorem and obtain $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ from $f(x,y,z)=0$ by without giving Simplification.
 - (2) If $y^3 - 3ax^2 + x^3 = 0$ then find $\frac{d^2 y}{dx^2}$.
- Que-2(A) Answer any one question out of two. (05)
- (1) If $f(x,y) = x^3 + xy + y^3$ then find the approximate value of $f(1.01, 2.98)$.
 - (2) Expand $\log(xy)$ in power of $(x-1)$ and $(y-1)$.
- Que-2(B) Answer any one question out of two. (05)
- (1) Find the greatest and smallest value of the function $f(x,y)=xy$ takes on ellips $\frac{x^2}{8} + \frac{y^2}{2} = 1$.
 - (2) Find the shortest distance from origin to the surface $xyz^2 = 2$.
- Que-3(A) Answer any one question out of two. (05)
- (1) In usual notations, derive radius of curvature for Cartesian curves.
 - (2) Find radius of curvature at any point on the cardiod $r = a(1 + \cos \theta)$.
- Que-3(B) Answer any one question out of two. (05)
- (1) Define radius of curvature and find radius of curvature at origin for the Curve $x^2+2xy+2y^2-4x = 0$.
 - (2) In usual notation derive radius of curvature for polar curves.
- Que-4(A) Answer any one question out of two. (05)
- (1) Obtain the oblique asymptotes of the general algebraic curve.
 - (2) Define concavity and convexity of the curve and also prove that $y = e^x$ is Concave upward everywhere.
- Que-4(B) Answer any one question out of two. (05)
- (1) Find the position and nature of double points of the curve $x^4-2ay^3-3a^2y^2-2a^2x^2+a^4 = 0$.
 - (2) Find all asymptotes to the curve $y^3-6xy^2+11x^2y-6x^3+x+y = 0$.
- Que-5(A) Answer any one question out of two. (05)
- (1) Define: Rectangular nbhd, circular nbhd, Limit of a function of two variables, Continuity of a function of two variables, Homogeneous function.
 - (2) if $u = x+y+z$, $uv = y+z$, $uvw = z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ and $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
- Que-5(B) Answer any one question out of two. (05)
- (1) Find radius of curvature at origin for the curve $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$.
 - (2) Show that origin is a point of inflexion for the curve $y = x^3$
